Exercises

1. Consider a unilateral care accident model in which the probability of an accident is given by $p(x)=e^{-x}$, where x is the level of injurer care, and e is the base of the natural logarithm. Let D be the dollar amount of damages suffered by the victim in the event of an accident, and let c be the unit cost of care for the injurer.

(a) Write down the expression for total accident costs (the injurer's cost of care plus expected damages), and solve for the optimal level of care, x^*, as a function of c and D. (Remember that $ln(e^x)=x$.)

(b) Show that x^* is increasing in D and decreasing in c.

(c) Calculate x^* for $D=500$ and $c=10$.

2. Consider the basic unilateral accident model where $p(x)$ is the probability of an accident, $p'<0$, $p''>0$; x is the injurer's spending on care; and D is the victim's *fixed* damages. Under a strict liability rule, the injurer faces liability, L, equal to D, and hence chooses efficient care of x^*.

(a) Suppose that the injurer has limited assets, A, to spend on liability. That is, $L \leq A$. How does this constraint affect the injurer's care choice when $A<D$? Show that care is increasing in A over this range.

(b) Now suppose that care involves an out-of-pocket expenditure, so that the asset constraint is now $x+L \leq A$, or $L \leq A-x$. Show that in this case, care may be greater than or less than x^*. Explain the result intuitively.

3. In the standard accident model, the risk of accidents often depends on the parties' activity levels as well as their care. Consider a model where accident risk depends on the *injurer's care* and the *victim's activity level*. For example, a pedestrian decides how often to walk on a busy street, and a driver decides how fast to drive. Let:

z = victim's activity level;
$B(z)$ = gross benefit of z to the victim, where $B'>0$, $B''<0$;
x = injurer's spending on care;
$L(x)$ = victim's expected damage per unit of z, where $L'<0$, $L''>0$.

(a) Derive the first order conditions describing the optimal activity and care levels.

(b) Show the actual choices of z and x under *no liability*, *strict liability*, and *negligence*. (Assume that the due standard under negligence is set at the efficient care level from part (a).) How do they compare to the social optimum?

4. Consider an accident setting in which an injurer can take care of x to reduce the probability of an accident, $p(x)$ ($p'<0$, $p''>0$). If an accident occurs, the victim's loss is $D(y)$, where y represents an expenditure made after the fact to *mitigate* the damages. Either the victim or the injurer can invest y.

(a) Derive the first order conditions characterizing the optimal levels of x and y. (Note that the optimum does not depend on which party chooses y.)

(b) Suppose initially that the injurer chooses both x and y. Derive the conditions describing the injurer's actual choice of these variables under *strict liability* and under *negligence* (where due care is set at the socially optimal x). Which rule is preferred?

(c) Now suppose that the victim chooses y while the injurer continues to choose x. Again derive the actual choices of these variables under strict liability and negligence. Which rule is preferred in this case?

5. Suppose that the gross demand for a risk product is given by $10-q$, and the producer's total cost of production is $q^2/2$, where q is quantity. Also suppose that the product creates a per unit risk of an accident equal to p, the damages per accident are D, and the producer's share of liability is s, $0 \le s \le 1$.

(a) Derive the efficient level of production of the product and show that the competitive market achieves that outcome regardless of s.

(b) Now suppose that the consumer perceives the risk of an accident to be αp, where $\alpha \neq 1$. Derive the market equilibrium as a function of s and α and show that q is only efficient if $s=1$ (i.e., if the producer is strictly liable). Also show that the sign of $\partial q/\partial s$ depends on whether α is less than or greater than one. Provide intuition for your answers.

6. The owner of a piece of undeveloped land agrees to sell it to a developer for a price P_0. The gross value of the land to the developer is $V(x)$, where x is an investment such that $V'>0$ and $V''<0$. Suppose that the developer must spend x *before* the transfer of ownership occurs (though after P_0 is set), and that once spent, x is sunk.

(a) Derive the condition for the socially optimal level of investment, x^*.

Suppose that when the transfer date arrives, the seller says that he will only complete the sale if the buyer agrees to a new price, P. Assume that P is determined by ordinary Nash bargaining at this point, where the gain from a successful transaction going forward is $V(x)-P$ for the developer and $P-P_0$ for the seller.

(b) Calculate P.

(c) Assuming that the developer correctly anticipated this later renegotiation, what level of investment, x, would he have initially chosen to maximize expected profit? How does it compare to the level from (a)?

(d) If the initial price, P_0, were costlessly enforceable, what level of x would the developer have chosen?

7. A landowner sells a piece of vacant land to a buyer for a price of $50,000. In the process of excavating the land for a development project, the buyer uncovers a cache of dinosaur bones that are worth $1millon to a museum. The seller sues to have the contract rescinded.

(a) Discuss the factors that the court should consider in ruling on this case. Comment in particular on the relevance of evidence, presented by the buyer, that he or she had consulted with an archeologist prior to the purchase.

(b) What if there were a federal law in place at the time of the sale saying that any archeologically important finds discovered on privately owned land belonged to the government and could be confiscated?

8. Consider a contract between a buyer and a seller, where:

V = value of performance to the buyer;
C = cost of performance to the seller (a random variable);
P = contract price, payable on performance.

Suppose that the realized value of C was unknown at the time the contract was made, but at the date of performance it turns out that $C>P$.

(a) Show that if bargaining between the buyer and seller is costless, the parties will agree on a price increase (or bonus) b such that performance occurs if and only if performance is efficient at the realized value of C.

(b) Derive the upper and lower bounds for b, assuming performance occurs.

(c) Assume that the parties split the surplus from renegotiation equally. Derive the resulting value of the bonus b.

9. Consider the following contract model:

V = value of performance to the buyer;
C = fixed cost of performance to the seller;
P = price, payable on performance, where $V>P>C$;

$q(x)$ = probability of performance;
x = seller's effort to avoid breach, where $q' > 0$, $q'' > 0$.

(a) Derive the socially optimal level of effort, x, by the seller.

(c) Show that *expectation damages* induce the seller to choose the socially optimal level of effort.

10. A landowner hires a tenant farmer to cultivate his land. The production function is $q=2\sqrt{x}$, where q is the expected crop yield and x is the farmer's effort (measured in dollars). Let p be the (fixed) market price for crops. Suppose that the contract between the landowner and the farmer specifies the farmer's compensation to be $\alpha+\beta pq$; that is, he receives a fixed payment α, and a share β of total revenue from sale of the crops, where $0 \leq \beta \leq 1$.

(a) Given this contract, write down the expected wealth of the farmer, U_F, and the landowner, U_L.

(b) Derive the socially optimal effort level of the farmer, x^*. (Hint: solve for the x that maximizes the joint wealth of the two parties, U_L+U_F.)

(c) Find the level of effort that maximizes the farmer's expected wealth, U_F, as a function of α and β. Denote it $\hat{x}(\alpha, \beta)$. How does it depend on these parameters?

(d) Compare the results from (b) and (c). What choices of α and β (if any) will make $\hat{x} = x^*$?

(e) Assume that the farmer will only enter the contract if $U_F \geq 0$. Use this along with your answer to (d) to find the combination of α and β that maximizes the landowner's expected wealth, given that the farmer is willing to participate. What is true of α? Interpret the results.

11. When property can be lost and recovered (e.g., straying cattle, shipwrecks), the economic problems concern incentives to avoid the original loss, and if the loss occurs, to recover the property. Let

y = dollar expenditure by the owner to prevent a loss;
V = value of the property in question;
$q(y)$ = probability of a loss, $q' < 0$, $q'' > 0$.

(a) Assume initially that recovery is not possible. Derive the condition for optimal prevention of loss. How does it vary with V?

(b) Now suppose that the owner can invest in recovery efforts following a loss. Specifically, let

x = dollar expenditure on recovery;
$p(x)$ = probability of a successful recovery, $p' > 0$, $p'' < 0$.

Derive the condition for optimal recovery effort and show how it varies with V. Also, reconsider the optimal investment in loss prevention. How does the possibility of recovery affect the owner's optimal investment in y, if at all? Explain.

(c) Finally, suppose that only someone *other than the owner* can invest in recovery (e.g., a professional treasure hunter). Assuming the same recovery technology, consider two rules for recovery: (1) Recovered property is restored to the original owner, and (2) Recovered property is owned by the finder (a finder-keepers rule). What choice of y will the original owner make, and what choice of x will the treasure hunter make, under each of these rules? Which rule is superior?

12. Consider a two period model of land development and regulation. An owner of undeveloped land can develop *now*, yielding a private benefit of V_1, or he can develop *later*, yielding a private benefit of V_2. Assume that $V_2 > V_1$, implying that waiting is privately optimal, all else equal.

Suppose that with probability p, development in either period will cause external damages of D (e.g., loss of the habitat of an endangered species), but with probability $1-p$, no harm will occur. Assume that $D > V_2$, so if the damage will occur, it is efficient to prohibit development. However, the government will only learn whether or not the damage will occur at the start of period two. If it learns that the damage will occur and the land is not yet developed, the government will prohibit development, but if the land was developed in period one, the government will not reverse the outcome.

(a) Show that from a social perspective it is always efficient for the landowner to wait until period two to develop.

(b) Let C be the amount of compensation paid to a landowner who did not develop in period one and is prohibited from developing in period two. Show that $C=0$ will result in an excessive incentive for the landowner to develop in period one. What value of C will induce the landowner to make the efficient decision with respect to the timing of development?

13. A potential criminal offender can commit a crime that imposes social harm of h. Assume that the private gain to the offender is g, where $g < h$. Society can potentially deter the offender by threatening to impose a sanction that would cost the offender s, but would cost society $c(s)$ to impose, where $c' > 0$. Assume that the enforcer knows the

magnitude of g, and that when indifferent between committing the crime and not committing the crime, the offender does *not* commit it.

(a) Assuming that society counts the offender's gains and costs in welfare, write down the expression for welfare as a function of s when (i) s is in the range where the offender is not deterred; (ii) s is in the range where the offender is deterred.

(b) What is the welfare maximizing level of s over each of the ranges in (a)?

(c) Use your answer from (b) to derive the condition under which it is efficient to deter the offender.

(d) How would your answers change if the offender's gains and costs were *not* counted in social welfare? Is it more or less likely that society would find it optimal to deter the offender?

14. Consider the optimal enforcement model in which only fines are available and the probability of apprehension is variable. Suppose that the gain to potential offenders is distributed uniformly on the interval $[0, \bar{g}]$, the cost of apprehension is $.5p^2$, the harm from an offense is h, and the offender's wealth is w. Given that the optimal fine in this case is $f^*=w$, derive an explicit expression for the optimal probability of apprehension, p^*. Show that p^* is increasing in h, decreasing in $\bar{g}$, and may be increasing or decreasing in w.

15. An injurer can take care to reduce the severity of accidents. Let the cost of care be c, which reduces damages from D_H to D_L, where we assume that

$$D_H > D_L + c \qquad (*)$$

If lawsuits are allowed, the victim can sue for damages at a cost k_v, where we assume that $k_v<D_L<D_H$. Thus, if allowed, the victim will always sue in the event of an accident. If a suit occurs, the injurer must also incur trial costs of k_i.

(a) If suits are allowed, what is the condition for the injurer to take care? Is it satisfied?

(b) Given your answer to (a), what are social costs if suits are allowed?

(c) Now suppose that suits are not allowed? Will the injurer take care? What are social costs in this case?

(d) Compare social costs with and without suits. What is the condition for suits to be socially desirable? How does the condition compare to (*)?

16. Repeat defendants like business firms often hire full time (in-house) attorneys rather than hiring an outside attorney as claims arise. The impact of an in-house attorney is to lower the variable cost of a trial at the expense of imposing fixed costs (the attorney's salary) on the defendant. This exercise asks you to use the asymmetric information model of settlement to derive the strategic effects of lowering the defendant's variable cost of a trial. To do so, modify the model as follows. Let:

αC_d = the defendant's cost of trial, where $\alpha \leq 1$.

All other aspects of the model remain the same.

(a) Derive the first order condition for the optimal settlement amount, $S^*(\alpha)$.

(b) Derive an explicit expression for $S^*(\alpha)$ for the case of the uniform distribution and show that $\partial S^*/\partial \alpha > 0$. Explain the result intuitively.

(c) Also for the uniform distribution, show that the probability of trial is decreasing in α. Explain the result.

(d) Let K be the fixed cost of hiring an in-house attorney. Compare the defendant's expected cost of hiring an in-house attorney ($\alpha<1$, $K>0$) versus an outside attorney ($\alpha=1$, $K=0$) as a function of α.

17. A prosecutor faces a population of defendants, some of whom are truly guilty and some of whom are truly innocent. Let P_G and P_I be the probabilities of conviction at trial of guilty and innocent defendants, respectively, and let s be the sanction at trial upon conviction. Finally, let C_d be the cost of a trial to the defendant.

(a) What is the highest plea offer that would just induce a guilty defendant to plead guilty? Will an innocent defendant accept this offer?

(b) What is the expected penalty (including trial costs, if any) for each type of defendant under this offer?

(c) What is the highest plea offer that would just induce an innocent defendant to plead guilty? Will a guilty defendant accept this offer?

(d) What is the expected penalty for each type of defendant under this offer?

(e) If the prosecutor's objective includes maximizing the expected penalty of guilty defendants and minimizing the expected penalty of innocent defendants, which of the two offers is preferred?

(f) Which offer involves lower trial costs for the prosecutor?

18. A potential offender enjoys a gain of g from committing an illegal act. The expected sanction upon conviction is s, but there is the possibility of legal error. In particular, if the offender commits the act, there is a probability r that he will *not* be punished, and if the offender did not commit the act, there is a probability q that he *will* be punished.

(a) Derive the condition for the offender to commit the act.

(b) Increasing the standard of proof for a criminal conviction will tend to lower q (the chance of wrongful conviction) but raise r (the chance of wrongful acquittal). How will this change affect the offender's decision from (a)?